



## Von Bertalanffy- Richard's Function for Modelling Height- Diameter Relationship in *Albizia lebbeck* L.

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### Abstract

The tree H-D relationship is frequently used as a key input variable in growth and yield modelling and to describe stand dynamics as well. The study aimed at establishing a powerful H-D model for *Albizia lebbeck* L. trees using von Bertalanffy- Richard's (vB- R's) function. A total of 42 trees were sampled from 5 (0.025 ha) circular plots with DBH ranging from 10.25- 91.33 cm. Total tree height also varied from 10.32- 25.03 m. Two, three and four parametric vB-R's functions were fitted to forty-two (42), H-D pairs of *Albizia lebbeck* L. trees to generate predictive height curves. Among the three model types, best fit model came out to be a 3- parametric model with least RSE, Mean Bias, AIC, RMSE and maximum Adj.- R<sup>2</sup> and it appeared to be biologically realistic too. Among the models tested, four parametric was quite close to three parametric model in describing biological significance of the H-D relationship but evaluating statistics of the 3- parametric function was slightly better than that of 4- parametric function, hence the former was considered for further biological interpretation. Moreover, 3 and 4 parametric models generated sigmoid- height curves (c, d>0), while 2- parametric model generated a close to concave shape (b close to 0). The discussed model functions can be applied to similar stand conditions from where study data was collected.

**Keywords:** *von Bertalanffy- Richard's function, H-D model, Albizia lebbeck L., sigmoid- height curves, concave shape*

### Introduction

Forests are one of the largest natural resources which affect life of virtually every person on this earth through its commercial and ecosystem services. In order to improve management practices, it requires continuous monitoring in terms of growth or increment. Forest growth is estimated by measuring stand density, basal area, timber volume, biomass and eventually growing carbon stocks. Moreover, forest management also demands monitoring of regeneration potential, mortality rate and survival analysis of the tree species. In order to predict forest dynamics through growth and yield simulation, two fundamental tree variables needed to be measured quickly and accurately which are namely Total height (TH) and Diameter at Breast Height (DBH).

Tree DBH for large number of trees can be measured accurately, quickly and at low

cost. But when it comes to tree height measurement in dense tropical forests, the entire process becomes costly, time consuming, labor- intensive and prone to observational and measurement errors. Measuring tree top in a dense and productive forest is always challenging, especially of tall trees due to several practical field inventory issues. Measurement of tree height is relatively complex, especially in forests with dense canopy and uneven terrain. However, measurement of small trees (up to 5 m height) is comparatively easy, which can be performed by pole method. But beyond 5 m height, instruments like hypsometer and clinometer are recommended whose working are based on the principle of geometry and trigonometry. *In situ* measurement of tree height in a dense forest can be time taking and difficult, as during

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DOI: <http://dx.doi.org/10.21746/aps.2021.10.12.5>

most measurements, the top of the individual trees cannot be viewed clearly. Thus, measuring tree height becomes a bottleneck in the process of conducting forest inventories.

This often, results in the exclusion of tree height as a variable for predicting forest growth. However, studies have shown that including tree height can significantly improve forest growth models because notable variations exist in tree height for a given stem diameter in tropical forests. To overcome this problem, height- diameter allometric relationships are often established which are mostly nonlinear in nature. Such models can be constructed from within a population (individual species) or from a community (mixed forest) and are available in both local or generalized forms. For developing H-D models, it is assumed that tree height is a function of diameter. Expression of model function goes like this:  $H(D)=f(D)$ , where H, is the tree height and DBH, is the tree diameter.

In general, H-D relationship of trees either takes a typical sigmoid or a concave curve shape. Sigmoid curve is generated when data is collected at an early stage i.e., of young trees, with monotonic increment ( $dH/dDBH > 0$ ), clear inflection point ( $d^2H/dDBH^2 = 0$ ) and an upper asymptotic value ( $H \rightarrow \text{asymptote as } DBH \rightarrow \square$ ). Concave curves also enjoy the above two mathematical properties of the S- shaped curve except for lack of inflection point. Such type of relationship is observed in datasets with older and larger trees beyond the inflection point.

Actually, there is no standard height- age or height- diameter relationship of trees because of the influence of both external and internal factors on DBH and height growth. However, under initial conditions of growth, a sigmoid curve describes the best relationship between H and D of a tree. The curve of the functional form represents a typical height cumulative growth curve, which starts at the original value, the growth rate increases uniformly to achieve maximum growth at an inflection point, and

afterwards gradually approaches some final asymptotic value which is determined by the genetic nature of the tree species and the carrying capacity of the given environment. On the other hand, a concave curve reflects an initial period of rapid growth and then the instantaneous growth rate monotonically decreases to some final asymptotic value. The concave curve growth model is a unique case in a 4- parametric vB-R's function and could be applied to simulate growth of fast-growing trees. In this case, both catabolic rate constant ( $\kappa$ ) and anabolic rate constant ( $\eta$ ) are greater than zero ( $\kappa$  and  $\eta > 0$ ).

While applying nonlinear models to H-D relationships, it should be kept in mind that the mathematical properties of the selected function i.e. model parameters and their relationships, should imitate the biological process of tree growth and the growth models selected should be meaningful and fulfill the biological interpretation criteria. The model parameters should have an inherent hypothesis related with source of function of the event described by the responsible variable.

In this study, the author has applied two, three and four parametric von Bertalanffy-Richards (vB- R's) function to predict total tree height of *Albizia lebeck* L. trees using DBH as the independent variable. The above model function has been widely used for simulating forest growth and yield. This model is highly valued for its accuracy and it has been utilized much more than any other function in studies of tree and stand growth (Lei and Zhang, 2004). Whether the growth model has a point of inflection or not mainly depends upon the shape parameter. In other words, whether the growth model demonstrates a sigmoid or a concave curve shape is based on different conditions of allometric shape parameters in the function. Thus, this dynamic model fulfills both data-related and reasonable biological criteria and not just data- related criteria.

The study site was selected at the foothills of central Kumaon which comprised two unique ecosystems namely bhabhar and terai. In contrast to bhabhar, terai region is

water logged alluvial plain with gentle south- east slope with deep and fertile moist loamy soil forming marshy land which are free from gravels and boulders (Mathur and Joshi, 2012).

In the present investigation, the author has tried to construct a local H-D model for *Albizia lebbeck* L. trees growing naturally in Lalkuan forest, in the terai region of central Kumaon Himalaya in Uttarakhand state of India. vB-R's model function (2, 3 and 4 parametric) was applied to predict total tree height using DBH as an independent variable. The author believes that the model discussed in this manuscript can be applied to predict tree height in the given tree species at a local level.

## Materials and Methods

### Site Details and Climatic Conditions

The field data was collected from Lalkuan forest between July 11- 13, 2021 which lies at the foothills and in terai region of district Nainital. Geographically, the area is located approximately between 29° 04' N, 079° 30' E with altitude ranging from 200 to 256 m asl. The area comprised of somewhat tropical and sub- tropical vegetation dominated by woody species namely. *Dalbergia sissoo*, *Acacia catechu*, *Albizia lebbeck* L., *Albizia amara* and *Albizia procera*. The site enjoys typical monsoon climate with rich humidity during July to mid- September. Nearly 70% of the annual rainfall in monsoonal during July to September. Mean maximum temperature is 22.5° C (January) and 45.3° C (June) and mean minimum temperature is 9.5° C (January) and 33° C (June) respectively.

### Data Collection and Sampling Procedure

5 circular plots of size 0.025 ha were randomly placed to sample forty- two (42) individual trees of *Albizia lebbeck* L. Stand density came out to be 336 stems/ha. Random sampling method was used for sampling trees. Plot method was used which is one of the most commonly used methods for all kind of vegetation sampling. The method is versatile and cost- effective too. In all, 5 circular plots (plot radius 8.92 m) were laid randomly, measuring the indicator parameter i.e. DBH and Total tree height.

Overall, 42 H-D pairs of *A. lebbeck* trees were measured from 5 sample plots. The study site had moderately dense vegetation, for which the size of the plot is apt (MacDickens, 1997). It was ensured that 5 plots cover the maximum range of DBH of trees along with corresponding height measurements. It was also ensured that a plot size accommodated, a minimum of 20 trees per plot. As it was a mixed- natural forest, so only *A. lebbeck* trees were taken into consideration while rest of other tree species were omitted from taking measurements within the study plots. All 42 H-D pairs of different diameter range were incorporated for nonlinear model fitting. Before model fitting the H-D pairs were arranged in ascending order of their respective DBH values. Initially, Girth at Breast Height (GBH) of trees (1.3 m from ground) was measured using an inch tape which was later converted to corresponding DBH values (Tripathi and Joshi, 2015). For height measurement, pole method was used for trees up to 5 m high. For trees above 5 m, clinometer was used.

Eventually, forty-two (42), H-D pairs were subjected to nonlinear regression analysis. Von Bertalanffy – Richard's function (vB-R's) was used for height curve fitting. Overall, 2, 3 and 4 parametric models were used to fit the Height- Diameter data which were later ranked on the basis of their predictive performances. The above function is dynamic, flexible and deals with both symmetric and asymmetric growth. Because of its dynamic nature, the model is used widely to deal with forest growth and modelling along with tree H-D relationships. Due to its flexible nature, the function produces acceptable curves under wide range of forestry modelling conditions.

### Model Development and Fitting

vB-R's model was selected for predicting Height-Diameter relationship based on literature review to fit the data set. It was assumed that the total tree height is a function of diameter. All model types had the following functional form:  $H_i = 1.3 + f(D_i, b) + \xi_i$

Where,  $H_i$  is the  $i$ th observation of the dependent variable i.e. tree height in meters,  $D_i$  is the  $i$ th observation of the independent variable i.e. DBH in cm,  $b$  is the vector of parameter to be estimated,  $\xi_i$  is the random error and  $f()$  is a nonlinear function, and  $i$  is the  $i$ th observation with  $i = 1,2,3\dots n$ . A constant term 1.3 is the height of the stem above ground at which DBH was measured. The term was added in the function to avoid prediction of zero  $H$  when  $D$  approaches zero.

All statistical calculations were performed in Microsoft Excel 2019 using *add in* functions viz. *Data Analysis tools*, "*Real Statistics Resource Pack*" and *Solver*. Different functional forms of the model were fitted by nonlinear least square (NLS) method using "*Solver*". It is designed on the powerful and reliable generalized reduced gradient (GRG) method and can be utilized as an easy iteration method to deal with nonlinear and asymptotic models (Okon *et al.*, 2021). Solver operates by fitting nonlinear regression function *via* an iterative algorithm (Bowen and Jerman, 1995) which minimizes the sum of squared error (SSE) between observed and predicted datasets. Thus, the model obtained maximum likelihood when SSE is minimized. For data fitting, number of iterations was set at hundred (100), with precision level of 0.000001. The convergence criterion for accepting the values of parameter estimates was taken as 0.001. The

smaller the convergence value, the more time solver takes to reach a solution. As the convergence value is reduced, precision of the model is improved.

#### Model Evaluation and Validation (Uncertainty Analysis of Model Parameters)

The three model types were evaluated on the basis of: performance criteria; significance of model parameters and practical application or biological significance of the model. Each of the model types were tested for their precision, accuracy, and reliability to predict height for *A. lebeck L* in a practical operational environment in terai region of Kumaon Himalaya. To do this, the author used the measured height of *A. lebeck L* trees and compared it with the heights estimated by each of these model types. The model performance was evaluated based on following criteria: Residual Standard Error (Chenge, 2021), Adjusted-  $R^2$  (Chenge, 2021), Mean Bias (Chenge, 2021), Akaike Information Criteria (Chenge, 2021), and Root Mean Square Error (Rivas *et al.*, 2014), followed by graphical presentations such as histograms, Q-Q plots of residuals and observed Vs predicted values of models. Shapiro- Wilk test was also performed for all the three model types to check for residual normality. Standardized residuals were also analyzed for any possible outliers in the dataset. Model with least RSE, Mean Bias, AIC, RMSE and maximum adjusted-  $R^2$  was considered to perform the best.

**Table 1:** Descriptive statistics of the data used in the study

	Variables	
	DBH (cm)	H (m)
Mean	33.407	18.702
SD	20.408	3.185
Range	10.25-89.93	10.32- 25.03
Kurtosis	1.5686	0.4281
Skewness	1.5773	0.3520
Outliers	No	No

Model validation was performed by Jackknife procedure (Harris, 1998). Model parameters were statistically tested to estimate the variance and bias i.e., uncertainties in their behavior. Jackknife method is basically a resampling technique

which involves a leave-one-out strategy of the estimation of parameters (mean), in a data set of "N" observations. In other words, if there are a total of "N" numbers in a data set, then the predictor is trained on N-1 training examples and tested on remaining

one data point, i.e., leave-one-out cross validation was performed. Then, the process was repeated for “N” times (42 data points) and the predicted values of each sample (model parameters) was calculated.

## Results

All parameter estimates of the three model types were significant ( $p < 0.05$ ). The three model types described more than 76% ( $R^2_{adj.} > 0.76$ ) of height variability (Table 3). The model type M2 showed the best fit (smallest RSE, Mean Bias, AIC, RMSE and largest  $R^2_{adj.}$ ). M2 was followed by M3 and then M1 i.e.,  $M2 > M3 > M1$ . All the model types showed more or less identical fit and validation statistics.

To identify the best model, graphical examination of fitted curves was also overlaid on observed data (Fig 1), along with standardized residual plots (Fig. 2) and QQ plots of residuals (Fig. 3). Moreover, frequency distribution of model parameters for the best fit model i.e., M2 was also performed using Jackknife resampling technique (Fig 4), to look for amount of uncertainty in the parameters. All the model parameters generated with different model types of vB-R function appeared to be biologically relevant and statistically significant ( $p < 0.05$ ).

All the residuals were found within 95% confidence limit and the frequency distribution (histogram) of model parameters (3- parametric) looked bell-shaped with symmetrical (normal) distribution (Fig 4). Frequency distribution of two (2) and four (4) parametric vB-R's function also followed a similar pattern. This hinted that there was no substantial heteroscedasticity problem with all the three model types. Figure 2 presented the plots of standardized residuals against the observed diameters of the fitting data. All the model types showed no systematic departure from random pattern. Residual analysis of M1, M2 and M3 at 95% confidence interval ( $\alpha = 0.05$ ) revealed that M2 is the best model (CI= 0.4225), followed by M3 (CI= 0.4238) and M1 (CI= 0.4633). Lower CI values indicate high accuracy or precision of data. The Shapiro- Wilk test of residuals of the three model types [M1-( $W = 0.9791$ ,  $p = 0.6331$ ) for fitting and ( $W = 0.9818$ ,  $p = 0.7312$ ) for validation; M2-( $W = 0.9561$ ,  $p = 0.0737$ ) for fitting and ( $W = 0.9524$ ,  $p = 0.0788$ ) for validation; M3-( $W = 0.9460$ ,  $p = 0.0617$ ) for fitting and ( $W = 0.9490$ ,  $p = 0.0595$ ) for validation] showed a p- value greater than 0.05, which suggested that residuals do not violate the assumption of normal random error.

**Table 2:** Model functions and parameters

Model	Function/Expression	Parameter	Source
vB- R (M1)	$H = 1.3 + a(1 - \exp^{-bD}) + \xi_i$	a, b > 0, a is the H final value (upper asymptotic value); b is the shape parameter	MacPhee <i>et al.</i> , 2018
vB- R (M2)	$H = 1.3 + a(1 - \exp^{-bD})^c + \xi_i$	a, b, c > 0, a is the upper asymptote; b is the growth rate or scale parameter and c is the shape parameter and inflection point	MacPhee <i>et al.</i> , 2018
vB - R (M3)	$H = 1.3 + a(1 - \exp^{-cD})^{1/(1-d)} + \xi_i$	a, b, c, d > 0, a is the upper asymptote; b is the biological constant; c is tree growth rate and d is the powerful exponent of tree height for anabolic rate which determines curve	Richards, 1959

		shapes and the location of an inflection point	
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\*1.3 is a constant used to account that DBH is measured at 1.3 m (in height from the ground); H= Total height of a tree in m; D= DBH in cm (1.3 m above ground); *exp* = exponential function; a - d are the regression parameters of the model;  $\xi_i$  = random error; for every model. M1, M2 and M3 are designated as model types. \*Column three (3) denotes biological interpretation of model parameters along with constrains. All functions are expressed in integral forms.

**Table 3:** Model fitting and evaluation statistics. \*RSE= Residual Standard Error; R<sup>2</sup>- Adj= Adjusted R<sup>2</sup>; e= Mean Bias; AIC= Akaike Information Criteria; RMSE= Root mean square error; 2P, 3P and 4P are number of parameters in the model

MODEL	RSE	R <sup>2</sup> -Adj	e	AIC	RMSE
vB-R (2P)	1.514	0.757	0.039227	38.839	1.551
vB-R (3P)	1.380	0.792	0.001096	33.064	1.432
vB-R (4P)	1.384	0.786	0.000026	35.322	1.455

Meyer function which is an exponential function, was also introduced to the fitted height data in order to compare for any concavity in fitting statistics of the three model types. It was observed that M1 behaved much more like a Meyer function (Meyer, 1940) with a slight tendency of a concave curve (b close to zero), than rest of the two models. The other two model types generated an S- shape curve (c and d >0).

Figure 1, depicted the H- D curve of *Albizia lebbbeck* L. trees. In M2, the height curve started at a fixed point and increased its instant growth rate until an inflection point

was reached, after that, the growth rate decreased to approach the stable asymptotic value of (25.976 m). Model M2 was selected for further analysis and biological interpretation as it performed best on basis of fitting and evaluation statistics. The upper asymptotic value of *A. lebbbeck* (M2) was calculated as 25.976 m. This is the maximum value of height determined by the productive capacity of the site. This, more or less falls in line with the fact that maximum height of *A. lebbbeck* trees in Kumaon foothills reaches up to 26.0 m (Jha, 1994).

**Table 4:** Uncertainty analysis of model parameters (model validation) using Jackknife resampling procedure. \*2P, 3P and 4P are number of parameters in the model; SE= Standard error; CI= Confidence Interval.

Model	Estimate/Parameter	Mean	SE	CI
vB-R (2P)	a	22.429	0.0203	0.0397
	b	0.056	0.0001	0.0002
vB-R (3P)	a	25.976	0.0739	0.1449
	b	0.017	0.0005	0.0011
	c	0.435	0.0011	0.0114
vB-R (4P)	a	24.394	0.0534	0.1047
	b	0.690	0.0036	0.0071
	c	0.023	0.0002	0.0005
	d	0.243	0.0014	0.0028

Moreover, c>0, signifies that the height curve generated from the function is an S- shape curve. Sigmoid curve has been widely applied in the field of forestry due to its flexibility, precision and meaningful analytical properties. Further, there was not much difference between M2 and M3 in

terms of model performance and biological interpretation.

#### Biological Significance of Model Parameters

The two parametric vB- R's function generated a close to concave shape (b = 0.056) with range of shape parameter (b ≤ 0).

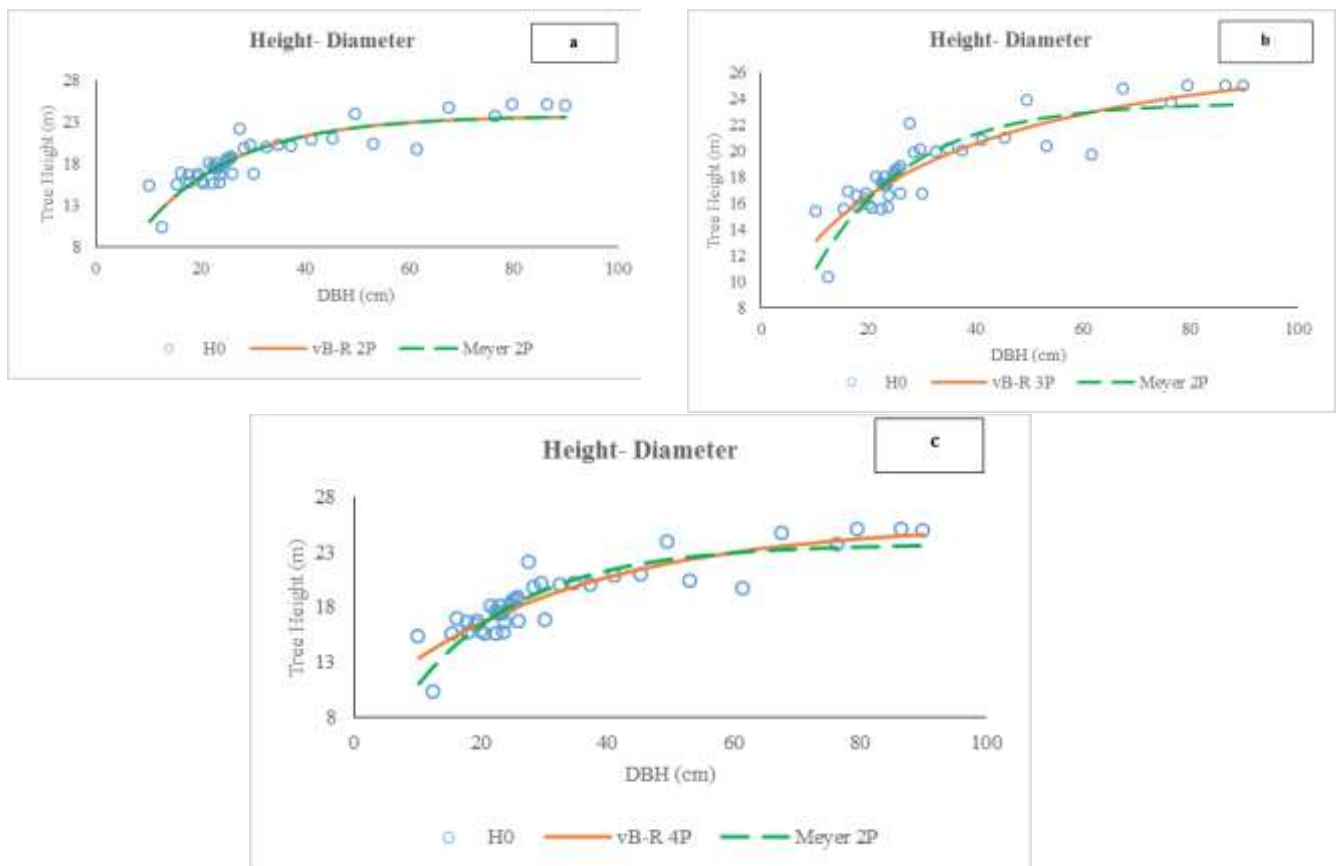
This function worked similar to two parametric Meyer function (or two parametric Mitscherlich function) when it comes to analyzing exponential H-D relationship of individual trees and stands. The three parametric vB-R's model produced a sigmoid curve ( $c > 0$ ) with inflection point and an upper asymptotic stable value. Two parametric model fitted the observed height data with least  $R^2_{adj}$  and maximum RSE, mean bias, AIC and RMSE hence was given last rank.

The four parametric functions offer the three most classical conditions of forest growth and yield kinetics. It covers a wide range of possibilities and here also the shape parameter ( $d$  in this case) decides the concave/sigmoid nature of the predictive curve. When  $d < 1$ , and  $\kappa$  and  $\eta > 0$ , then the curve will take a sigmoid shape with inflection point and an upper asymptote. This is the most common curve in height-diameter growth modelling and it also fitted well to *A. lebbeck* H-D data. If  $d > 1$ , and  $\kappa$  and

$\eta < 0$ , then the function also produces a sigmoid curve with inflection point and an asymptote. This type of conditions is suitable for slow growing trees and stands. Lastly, when  $d < 0$ , and  $\kappa$  and  $\eta > 0$ , then the curve takes a concave shape with no inflection point but an upper asymptote. When,  $d = 0$ , the function behaves like Meyer or Mitscherlich function. Here  $\kappa$  and  $\eta$  denotes catabolic and anabolic rate constants.

### Discussion

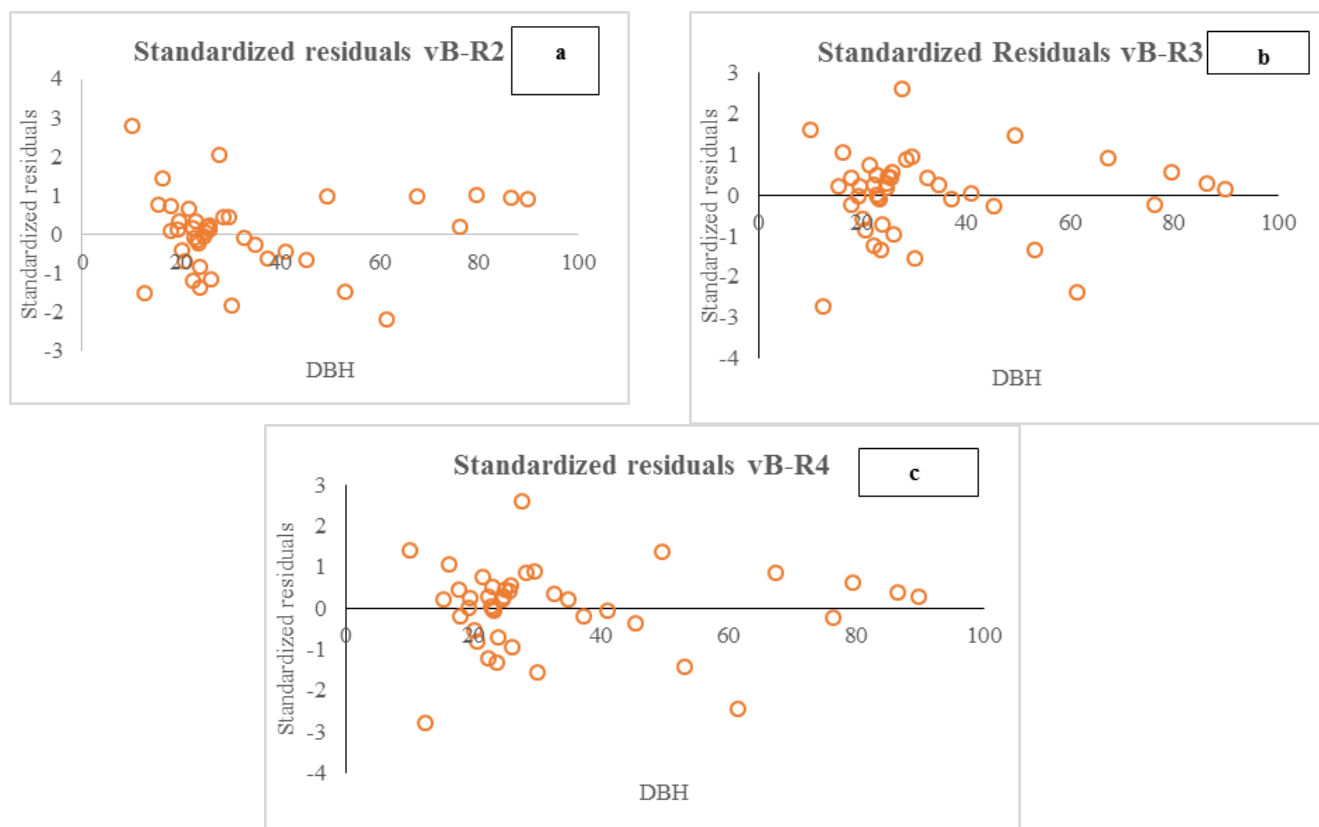
After analyzing the observed height data using vB-R's function, it can be said that all the three model types performed reasonably well in fitting the H-D data for 42 *A. lebbeck* trees sampled randomly from lalkuan forest. The evaluation of models was based on multiple performance criteria. When selecting a functional form for H-D relationship, following characteristic of the function should be kept in mind: mathematical properties, biological interpretation of model parameters and precision (accuracy) of the model.



**Figure 1 (a), (b), (c):** Height- Diameter curve fitting along with Meyer curve. \*H0 is the observed height in meter and vB-R and Meyer are fitted heights obtained from the three model types; 2P, 3P and 4P signifies number of parameters in the model function

Although, at a given moment, any model may be considered superior or inferior in a specific situation, but in general vB-R's function has proven to be more effective in describing H-D relationship of tree species in a natural forest or plantations. In *A. lebeck* also, the function was suitable in explaining and interpreting H-D correlations on basis of low RSE, mean bias, AIC, RMSE and high  $R^2_{adj}$  values. All the three model types fitted well to the H-D data which once again proved model flexibility and versatility in

explaining dynamic biological phenomena. In addition, the relationship between tree height and diameter is best described by sigmoid curves, where the tree height generally grows with increase of DBH and then stabilize. Thus, we see that the three and four parameter models produced a sigmoid curve and substantial information regarding the biological event (H- D correlation) and growth kinetics of *A. lebeck* trees could be gathered.



**Fig 2: (a), (b) and (c):** Standardized residuals analysis plots of (3) H-D relationship models fitted for *Albizia lebeck* l. \*vB-R2, vB-R3 and vB-R4 are von Bertalanffy- Richard's two parametric, three parametric and 4 parametric models-

Among the two, three and four parametric models it was found that the three parametric function is often more reliable for forestry models. However, the three parametric models sometimes suffer from problems regarding model convergence and non- positive definite random- effect variance- covariance matrix (Mehtatalo, et

al., 2015). Such type of issue was not found while fitting *A. lebeck* height data.

The discussed results also fall in line with the fact that three parametric model function is superior than other two model types in explaining H- D relationship of trees. Nevertheless, two- parametric model

functions are easy to fit and are quick to achieve convergence in most situations.

Thus, it can be stated that the above two functional types (M2 and M3) are sigmoidal in nature with an upper asymptote that can describe the H-D correlation phenomenon in

a more holistic way. Therefore, it is suggested that three and four parametric model types could be applied for predictions of the H-D relationship for *A. lebeck* species growing naturally in lalkuan forest, at the foothills of central Kumaon Himalaya.

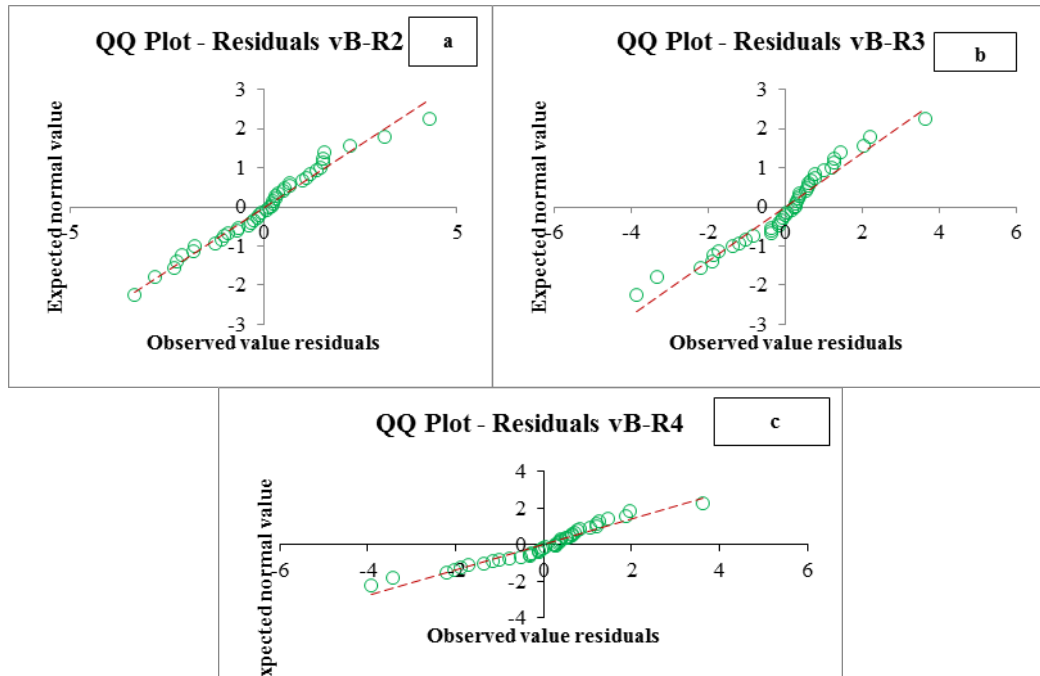


Figure 3 (a), (b) and (c) are the Q-Q plots of residuals

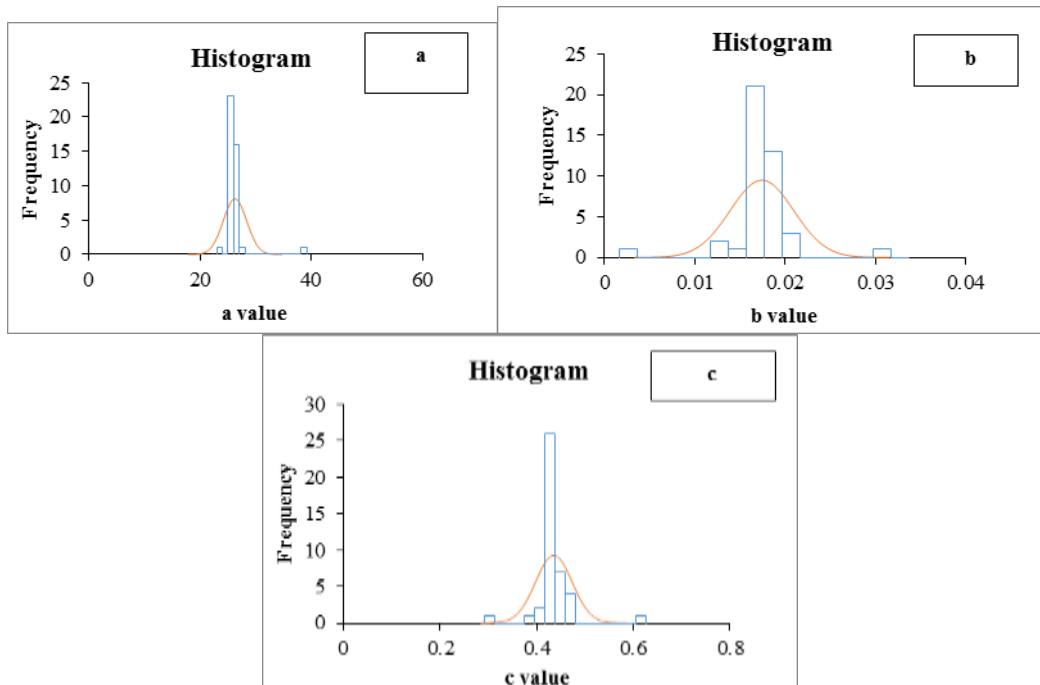


Figure 4: Frequency distribution of model parameters a, b and c obtained from Jackknife resampling procedure for the best fit (3 parametric) model. The height bars indicate the number of parameter values in the equally spaced bins. The curve is a theoretical normal distribution.

**Conclusion**

Routine measurement of diameter and height of individual trees is a common

practice in forest mensuration. Measuring height and diameter of individual trees is prerequisite for designing and developing complex forest inventory programs. Regarding forest inventories, measuring tree height is a difficult task. Moreover, omission of tree height from the function results in poor model fitting and performance. In order to deal with this issue, H- D allometric models are often developed where the nonlinear relationship between tree height and the diameter is utilized to predict total tree height of individual trees and stands. The present investigation could help forest managers to reduce cost and save time during the monitoring of *Albizia lebbek* L. trees growing in Lalkuan forest and eventually take precise decisions in forest management and planning.

#### Acknowledgements

The author wish to thank the Head of the Department of Plant Biology, Kumaon University, Almora for providing all necessary facilities during the course of the study. Prof. Hema Joshi's timely and preliminary review greatly strengthened the clarity of the manuscript. Assistance provided by the technical staff during the field survey and data collection is highly appreciated. Finally, thanks to anonymous reviewers for their suggestions and constructive comments.

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**Source of support:** Nil; **Conflict of interest:** Nil.

**Cite this article as:**

Tripathi, M. "Von Bertalanffy- Richard's Function for Modelling Height- Diameter Relationship in *Albizia lebbeck* L.". *Annals of Plant Sciences*.10.12 (2021) pp. 4401-4411. DOI: <http://dx.doi.org/10.21746/aps.2021.10.12.5>